

1. Consider sorting  $n$  numbers sorted in array  $A$  by first finding the smallest element of  $A$  and exchanging it with the element in  $A[1]$ . Then find the second smallest element of  $A$ , and exchange it with  $A[2]$ . Continue in this manner for the first  $n - 1$  elements of  $A$ . This is *selection sort*. Write pseudocode for this algorithm, and analyze its best-case and worst-case runtime in asymptotic notation.
2. Place the following functions into increasing asymptotic order. If two or more of the functions are of the same asymptotic order, then indicate this. Prove the correctness of your ordering. (In other words, if you claim that  $g(n)$  is greater than  $f(n)$  then show that  $f(n) \in O(g(n))$  but  $f(n)$  is not in  $\Theta(g(n))$ ).

$$4n, \quad n^2, \quad n \log n, \quad n \ln n, \quad \lg n, \quad e^n$$

Note:  $\log$  means logarithm base 2, and  $\ln$  means logarithm base  $e$  (natural log).

3. Let  $f(n)$  and  $g(n)$  be asymptotically positive functions. Prove or disprove the followings in few sentences.
  - $f(n) + o(f(n)) = \theta(f(n))$
  - $f(n) = \theta(f(n/2))$
  - $f(n) = O(g(n))$  implies  $2^{f(n)} = O(2^{g(n)})$
4. Give asymptotic upper and lower bounds for  $T(n)$ .
  - $T(n) = 4T(n/3) + n \log n$
  - $T(n) = 3T(n/3 - 2) + n/2$
  - $f(n) = \sqrt{n}T(\sqrt{n}) + n$