Due by Feb 9, 11:59 pm.

1. (30 points) Consider the *Permute-By-Sorting(P)* algorithm we discussed in class.

Algorithm 1 Permute-By-Sorting(P)

$n = length(P)$
For $i = 1$ to n
$key[i] = RANDOM(1, n^3)$
Sort P, using key as sort keys.
Return P

- Prove that the probability that all keys are unique is at least $1 - 1/n$.
- Show that the probability of obtaining *any* permutation is $1/n!$.

2. (40 points) Consider n cities $C = [c_0, \cdots c_{n-1}]$, where all adjacent cities are connected by a road, i.e. c_i is connected to c_{i+1}. The distance between any two cities c_i and c_j is $|i - j|$. We want to install the minimum number of electric towers in these cities such that all cities receive electric power. We can build at most one electrical tower in every city, and each tower can provide electricity to all neighboring cities at a distance at most k from the tower.

   Find the minimum number of towers we must build such that all cities receive electric power.

3. (30 points) Let $A[1..n]$ be an array of n distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called an inversion of A. Suppose that the elements of A form a uniform random permutation of $[1, 2, \cdots, n]$. Use indicator random variables to compute the expected number of inversions.