Due by Feb 16, 11:59 am.

1. (30 points) In class, we showed that the expected length of the longest streak of heads in n coin flips is upper bounded by $O(\log n)$. Prove that it is lower bounded by $\Omega(\log n)$.

2. (20 points) How many people should be invited to a party in order to make it likely that there are *three* people with the same birthday?

3. (50 points) Consider the coupon collector's problem: There are n types of coupons, and we articipate in a series of independent trials, where on each trial we have equal probability $(1/n)$ of getting each coupon. Let X denote the number of trials we expect to partake in before we collect all coupons.

   - (10 points) Use Markov's inequality to bound $Pr(X \geq 2n \ln n)$.
   - (20 points) Use Chebyshev's inequality to bound $Pr(X \geq 2n \ln n)$.
   - (20 points) The union bound states that "if $X_1, \cdots, X_t$ are t possibly dependent random events, then the probability that all events occur is at least $1 - \sum_{i=1}^{t} (1 - pr(X_i))$." Using union bound show that after $2n \ln n$ trials all coupons are collected with probability at least $1 - 1/n$.