Spring 2018 **CS 513: # 5 Fast Fourier Transform** Ghashami, Kleindessner

Due by February 23, 23:59 pm.

Exercise 1 (10 + 15 points)

a) What is the DFT of $(1, 0, 0, 0) \in \mathbb{C}^4$? And of which vector is $(1, 0, 0, 0)$ the DFT?

b) Apply RECURSIVE-FFT (version of the lecture or CLRS Chapter 30.2) to $(1, 0, 1, -1) \in \mathbb{C}^4$. Provide the final output as well as all intermediate steps in the execution.

Exercise 2 (10 points)

Prove that n point-value pairs (with distinct points) are necessary to uniquely specify a polynomial of degree smaller than n, that is show that for any distinct $x_0, \ldots, x_{n-2} \in \mathbb{C}$ and any $y_0, \ldots, y_{n-2} \in \mathbb{C}$ there exist at least two different polynomials $A(x)$ and $B(x)$ of degree smaller than n with

$$A(x_j) = B(x_j) = y_j, \quad j = 0, \ldots, n-2.$$

Exercise 3 (30 points)

Let $A(x) = \sum_{j=0}^{n-1} a_j x^j$ and $x_0 \in \mathbb{C}$. Show that there exists a unique polynomial $Q(x)$ of degree smaller than $n-1$ and a unique $r \in \mathbb{C}$ such that

$$A(x) = Q(x) \cdot (x - x_0) + r.$$

Provide the pseudocode of an algorithm that computes the coefficients of $Q(x)$ and r with running time $\mathcal{O}(n)$.

Exercise 4 (5 + 30 points)

a) Given distinct $x_0, \ldots, x_{n-1} \in \mathbb{C}$ and arbitrary $y_0, \ldots, y_{n-1} \in \mathbb{C}$, the interpolation problem requires to find a polynomial $P(x)$ of degree smaller than n such that $P(x_j) = y_j$, $j = 0, \ldots, n-1$. Show that the solution to this problem is given by

$$P(x) = \sum_{k=0}^{n-1} y_k \frac{\Pi_{j \neq k}(x - x_j)}{\Pi_{j \neq k}(x_k - x_j)}. \tag{1}$$

b) Provide a strategy that allows us to find the coefficients of $P(x)$ as given in (1) with running time $\mathcal{O}(n^2)$.

Hint: Find the coefficients of $\Pi_{j \in \{0,\ldots,n-1\}}(x - x_j)$ *and make use of Exercise 3. You may make use of Exercise 3 even if you have not solved it.*