Due by March 23, 23:59 pm.

Exercise 1 (10 points)

Show that for any concrete decision problem in NP there exists an algorithm A and a polynomial p such that A solves the problem in time $\mathcal{O}(2^{p(n)})$, where n is the input size.

Exercise 2 (15 + 10 points)

a) Prove that if NP $\neq$ co-NP, then P $\neq$ NP.

b) Prove that if NP $\neq$ co-NP, then NPC $\cap$ co-NP $= \emptyset$.

Exercise 3 (10 + 10 + 15 + 30 points)

The **Vertex-cover-problem** is the following decision problem: given an undirected graph $G = (V, E)$ and an integer k, does there exist $S \subseteq V$ with $|S| \leq k$ such that for every edge of G at least one of its endpoints is in S (such S is called a vertex cover)?

The **Clique-problem** is the following decision problem: given an undirected graph $G = (V, E)$ and an integer k, does there exist $S \subseteq V$ with $|S| \geq k$ such that any two distinct vertices in S are connected by an edge in G (such S is called a clique)?

The **Set-cover-problem** is the following decision problem: given a universe $U = \{x_1, \ldots, x_n\}$, a collection $S_1, \ldots, S_m$ of subsets of U, that is $S_i \subseteq U$, $i = 1, \ldots, m$, and an integer k, does there exist $L \subseteq \{1, \ldots, m\}$ with $|L| \leq k$ such that $\cup_{l \in L} S_l = U$?

The **Steiner-tree-problem** is the following decision problem: given an undirected graph $G = (V, E)$, a subset $T \subseteq V$ (we call the vertices in T terminals) and an integer k, does there exist $H \subseteq E$ with $|H| \leq k$ such that for all $t_1 \neq t_2 \in T$ there exists a path in G that connects t_1 and t_2 and only uses edges in H?

a) Show that the Vertex-cover-problem is in NPC.

b) Show that the Clique-problem is in NPC.

c) Show that the Set-cover-problem is in NPC.

d) Show that the Steiner-tree-problem is in NPC.